Basic Level of Concepts in Formal Concept Analysis

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Abstract. The paper presents a preliminary study on basic level of concepts in the framework of formal concept analysis (FCA). The basic level of concepts is an important phenomenon studied in the psychology of concepts. We argue that this phenomenon may be utilized in FCA for selecting important concepts. In the other direction, we argue that FCA provides a simple framework for studying the phenomenon itself. In this preliminary study, we attempt to draw the attention of researchers in FCA to the basic level of concepts. Furthermore, we propose a formalization of one of the several existing psychological views of the basic level, present experiments, and outline future research in this direction.

Keywords: formal concept analysis, psychology, concepts, basic level

1 Introduction

1.1 Motivation

It is a well-known fact that, usually, a concept lattice contains a considerable number of formal concepts. When assessed by domain experts, some of the concepts are found more important (or natural) than others. This observation may be utilized for selecting only some formal concepts—the important ones—and filtering out the others. In the literature, one may find several approaches that implement this general idea of selecting important concepts. In this paper, we propose and approach based on a phenomenon well-known in the psychology of concepts, namely the basic level of concepts. In addition to the aim of utilizing the basic level of concepts in FCA, we would like to draw the attention of researchers in FCA to this phenomenon as well as to argue that FCA may be seen as a simple formal framework for studying this phenomenon itself and thus be of interest for the psychologists of concepts (see also [1]).

1.2 Paper Overview

In Sections 1.3 and 1.4, we provide an overview of related work and preliminaries from FCA. In Section 2, we describe the phenomenon of the basic level of concepts and provide main references to the work in the psychology of concepts. An approach to formalize one psychological view of the basic level of concepts is presented in Section 3. Examples and experiments which demonstrate this approach are provided in Section 4. In Section 5 we conclude the paper and outline future research.

1.3 Related Work

The most relevant to this paper is the work on the stability of a formal concept and other indices that seek to assign indices to formal concepts representing the importance of formal concepts [14, 15]. In the approaches mentioned in the foregoing paragraph, no other information is used to compute the indices than the one contained in the formal context (or the concept lattice, which is uniquely determined by the formal context). Another idea, initiated in [2] and later developed in [3, 4], proposes to utilize the background knowledge to select important concepts. Other approaches to selection of only certain formal concepts have been proposed in [7, 8, 16, 23].

1.4 Preliminaries and Notation

We assume that the reader is familiar with basic notions of formal concept analysis [11]. A formal context is denoted by $\langle X, Y, I\rangle$. Formal concepts of $\langle X, Y, I\rangle$ are denoted by $\langle A, B\rangle$. A pair $\langle A, B\rangle$ consisting of $A \subseteq X$ and $B \subseteq Y$ is called a formal concept if and only if $A^{\uparrow} = B$ and $B^{\downarrow} = A$ where

$$A^{\uparrow} = \{y \in Y \mid \text{ for each } x \in X : \langle x, y\rangle \in I\},$$
$$B^{\downarrow} = \{x \in X \mid \text{ for each } y \in Y : \langle x, y\rangle \in I\}$$

are the set of all attributes common to all objects from A and the set of all objects having all the attributes from B, respectively. The set of all formal concepts of $\langle X, Y, I\rangle$ is denoted by $\mathcal{B}(X, Y, I)$. $\mathcal{B}(X, Y, I)$ equipped with a subconcept-superconcept partial order $\leq$ is the concept lattice of $\langle X, Y, I\rangle$.

2 Basic Level of Concepts in the Psychology of Concepts

The psychology of concepts is a field in cognitive psychology studying human concepts and their cognitive role. The psychology of concepts has been developed systematically since 1950s, but one may find related work much before the 1950s, with [10] being probably the first work using what has become the most common approach to experimental studies of concepts [18]. A comprehensive overview of the main issues involved in the psychology of concepts is given in [19] which is our main source in this paper.

An important place in the studies in the psychology of concepts is occupied by concept hierarchy, i.e. a particular way of organizing concepts using the superordinate-subordinate (superconcept-subconcept) relationship. An extensive experimental work has been done on various issues surrounding concept hierarchies. One of them, central to this paper, is the basic level of concepts.

The basic level of concepts is known from ordinary life. When we see a particular dog, say a German Shepherd named Alex, we say "This is a dog," rather than "This is a German Shepherd" or "This is a mammal." That is, to name the object, we use a particular concept that we somehow prefer to other concepts. In this case, we prefer the concept of a dog to the concept of a German Shepherd and to the concept of a mammal. The preferred concepts are called the concepts of the basic level.[1]

In the psychology of concepts, the basic level of concepts is intuitively understood as

> "...the most natural, preferred level at which to conceptually carve up the world. The basic level can be seen as a compromise between the accuracy of classification at a maximally general level and the predictive power of a maximally specific level." [19, p. 210].

[5] is the first work suggesting that people consistently use a kind of a middle level concepts in speach. Since then, several studies in basic level have been conducted. For example, it has been observed [19, Chap. 7] that people across cultures tend to use the same level of concepts when naming plants and animals (the level corresponds to genus). In addition, it has been observed experimentally that people with less knowledge about a domain tend to use more general concepts while people with extensive domain knowledge tend to use more specific concepts as the basic level concepts.

For our purpose, it is important to note that there exist several informal definitions, of the basic level. Some of them have been developed by Eleanor Rosch [20, 21]. The one we attempt to use and formalize within FCA in this paper is based on experimental work according to which the basic level is distinguished by the number of common attributes. According to these experiments, when people are asked to list attributes of a concept from the basic level and its superordinate and subordinate concepts, the following pattern may be observed. Only a few attributes are listed for the concepts of a superordinate level, while many more attributes are listed for the concepts of the basic level and the subordinate concepts. Moreover, the number of attributes listed for the subordinate concepts is only slightly larger than the one for the basic level. In addition, the experiments

[1] Related is our experience from explaining FCA. It is quite common that people not familiar with FCA ask questions like: "Given a particular object, what is the (appropriate) formal concept to which the object belongs?" To people familiar with FCA, such question may suggest that the person asking does not understand FCA well yet, because there are usually several formal concepts that cover a particular object, the most specific of them being the object concept generated by the given object. Nevertheless, such question may probably be seen as a manifestation of the phenomenon of the basic level of concepts which is inherently present in human reasoning with concepts: The person is asking for a concept from the basic level.

show that the nature of the attributes plays a role as well. Namely, for superordinate concepts, mostly functional attributes were listed, such as "keeps you warm"; for basic level, people listed nouns and adjectives as properties, such as buttons, belt loops; for subordinate concepts, additional adjectives were listed, such as those referring to color. In [21], the authors concluded what may be considered a generalized view of the above-described observation: The objects of the basic level concepts are similar to each other, the objects of the superordinate concepts are significantly less similar, while the objects of the subordinate concepts are only slightly more similar. This way of characterization of the basic level is utilized in Section 3.

Due to the lack of space in this paper, we do not comment in detail on the other psychological views of the basic level. Let us only note that the literature contains studies of several interesting aspects of the basic level, including the role of the basic level in cognitive processes such as the speed of classification or predictive capability. Let us also note that many studies were performed within specific domains and that the basic level was described using domain-specific criteria such as types of movements or visual characteristics of the objects.

3 An Approach to Basic Level in FCA

In this section, we propose an approach to identifying concepts of a basic level in a concept lattice inspired by Rosch's definition of basic level concepts described in Section 2. Informally, we call a cohesion of a concept a measure of whether the objects to which that concept applies are pairwise similar. According to Rosch's definition, a formal concept $\langle A, B\rangle$ belongs to the basic level if it satisfies the following properties:

(BL1) $\langle A, B\rangle$ has a high cohesion.
(BL2) $\langle A, B\rangle$ has a significantly larger cohesion than its upper neighbors.
(BL3) $\langle A, B\rangle$ has only a slightly smaller cohesion than its lower neighbors.

Note that the upper neighbors of $\langle A, B\rangle$ are the concepts that are more general than $\langle A, B\rangle$ and are directly above $\langle A, B\rangle$ in the hierarchy of concepts. The lower neighbors are defined analogously. The sets of all upper and lower neighbors of c (i.e. elements covering c and covered by c) is denoted by $\mathcal{UN}(c)$ and $\mathcal{LN}(c)$, respectively. That is,

- $\mathcal{UN}(c) = \{d \in \mathcal{B}(X, Y, I) \mid c < d$ and there is no d' for which $c < d' < d\}$,
- $\mathcal{LN}(c) = \{d \in \mathcal{B}(X, Y, I) \mid c > d$ and there is no d' for which $c > d' > d\}$.

Furthermore, we use the following notation and its variants:

- $sim(x_1, x_2)$ denotes the degree (or index) of similarity of objects x_1 and x_2.
- $coh(c)$ denotes the degree (or index) of cohesion of formal concept c.

Similarity of objects x_1 and x_2 can naturally be assessed by similarity of their corresponding intents, i.e. by similarity of $\{x_1\}^{\uparrow}$ and $\{x_2\}^{\uparrow}$. That is, given an appropriate similarity measure sim_Y with $sim_Y(B_1, B_2)$ being a similarity degree of sets $B_1, B_2 \subseteq Y$ of attributes, we may put

$$sim(x_1, x_2) = sim_Y(\{x_1\}^{\uparrow}, \{x_2\}^{\uparrow}). \tag{1}$$

In our experiments, we used the following well-known functions for sim_Y:

$$sim_{\mathrm{SMC}}(B_1, B_2) = \frac{|B_1 \cap B_2| + |Y - (B_1 \cup B_2)|}{|Y|}, \tag{2}$$

$$sim_{\mathrm{J}}(B_1, B_2) = \frac{|B_1 \cap B_2|}{|B_1 \cup B_2|}. \tag{3}$$

Note that (2) is the simple matching coefficient and that (3) is the Jaccard index [22]. sim_{SMC} is the number of attributes on which B_1 and B_2 agree (either $y \in B_1$ and $y \in B_2$, or $y \notin B_1$ and $y \notin B_2$) divided by the number of all attributes. sim_{J} is the number of attributes that belong to both B_1 and B_2 divided by the number of all attributes that belong to B_1 or B_2. That is, while sim_{SMC} treats both presence and non-presence of attributes symmetrically, sim_{J} disregards non-presence. This is the main conceptual difference between sim_{SMC} and sim_{J}.

A simple approach to measure the cohesion $coh(A, B)$ for a formal concept $\langle A, B\rangle \in \mathcal{B}(X, Y, I)$ is the following:

$$coh^{\varnothing}(A, B) = \frac{\sum_{\{x_1, x_2\} \subseteq A, x_1 \neq x_2} sim(x_1, x_2)}{|A| \cdot (|A| - 1)/2}. \tag{4}$$

That is, $coh^{\varnothing}(A, B)$ is the average similarity of two objects covered by the formal concept $\langle A, B\rangle$. Alternatively, we might put

$$coh^{\mathrm{m}}(A, B) = \min_{x_1, x_2 \in A} sim(x_1, x_2), \tag{5}$$

in which case the cohesion is the least similarity degree of any two objects covered by $\langle A, B\rangle$.

We are now going to assign to every formal concept $\langle A, B\rangle$ of $\langle X, Y, I\rangle$ a degree $BL(A, B)$ to which $\langle A, B\rangle$ is a concept from the basic level. Given that the concepts from the basic level need to satisfy conditions (BL1), (BL2), and (BL3), it seems natural to construe $BL(A, B)$ as the degree to which a conjunction of the three propositions, (BL1), (BL2), and (BL3), is true. That is, to put

$$BL(A, B) = \mathcal{C}(\alpha_1(A, B), \alpha_2(A, B), \alpha_3(A, B)), \tag{6}$$

where

- $\alpha_i(A, B)$ is the degree to which condition (BLi) is satisfied, $i = 1, 2, 3$,
- $\mathcal{C}$ is a "conjunctive" aggregation function; that is, if propositions φ_1, φ_2, and φ_3 have truth degrees α_1, α_2, and α_3, respectively, then the conjunction φ_1 *and* φ_2 *and* φ_3 has the truth degree $\mathcal{C}(\alpha_1, \alpha_2, \alpha_3)$.

A simple form of $\mathcal{C}$ is obtained by taking a t-norm [13] $\otimes$ and to put

$$\mathcal{C}(\alpha_1, \alpha_2, \alpha_3) = \alpha_1 \otimes \alpha_2 \otimes \alpha_3.$$

We assume that the truth degrees are numbers in $[0, 1]$ and use the product (Goguen) t-norm, given by $a \otimes b = a \cdot b$, in the experiments.

For $\alpha_1(A, B)$, $\alpha_2(A, B)$, and $\alpha_3(A, B)$, the following definitions seem natural choices (coh^* denotes $coh^{\varnothing}$ or coh^{m}, see (4) and (5)):

$$\alpha_1^*(A, B) = coh^*(A, B), \tag{7}$$

$$\alpha_2^{\varnothing *}(A, B) = 1 - \frac{\sum_{c \in \mathcal{UN}(A,B)} coh^*(c)/coh^*(A, B)}{|\mathcal{UN}(A, B)|}, \tag{8}$$

$$\alpha_2^{\mathrm{m}*}(A, B) = 1 - \max_{c \in \mathcal{UN}(A,B)} coh^*(c)/coh^*(A, B), \tag{9}$$

$$\alpha_3^{\varnothing *}(A, B) = \frac{\sum_{c \in \mathcal{LN}(A,B)} coh^*(A, B)/coh^*(c)}{|\mathcal{LN}(A, B)|}, \tag{10}$$

$$\alpha_3^{\mathrm{m}*}(A, B) = \min_{c \in \mathcal{LN}(A,B)} coh^*(A, B)/coh^*(c). \tag{11}$$

Remark 1. Let us explain the meaning of formulas (7)–(11). The formulas are designed so that the values of $\alpha_1(A, B)$, $\alpha_2(A, B)$, and $\alpha_3(A, B)$ (and their variants given by the superscripts) may naturally be interpreted as the truth degrees to which the propositions in (BL1), (BL2), and (BL3) are true.

Ad (7): Clearly, (7) may be interpreted as the truth degree of (BL1).

Before discussing the other formulas, let us observe that if $coh^*(c_1) \leq coh^*(c_2)$, then $\frac{coh^*(c_1)}{coh^*(c_2)} \in [0, 1]$ may be interpreted as the truth degree of "$coh^*(c_1)$ is only slightly smaller than $coh^*(c_2)$", and hence $1 - \frac{coh^*(c_1)}{coh^*(c_2)} \in [0, 1]$ may be interpreted as the truth degree of proposition "$coh^*(c_1)$ is significantly smaller than $coh^*(c_2)$". Assume therefore that in the factions $\frac{coh^*(c_1)}{coh^*(c_2)}$ in (7)–(11), we always have $coh^*(c_1) \leq coh^*(c_2)$.

Ad (8): Since

$$\alpha_2^{\varnothing *}(A, B) = \frac{\sum_{c \in \mathcal{UN}(A,B)} 1 - coh^*(c)/coh^*(A, B)}{|\mathcal{UN}(A, B)|},$$

it follows that $\alpha_2^{\varnothing *}(A, B)$ may be interpreted as the truth degree of "on average, the upper neighbors of $\langle A, B\rangle$ have a significantly smaller cohesion than $\langle A, B\rangle$", which is one possible meaning of (BL2).

Ad (9): Since

$$\alpha_2^{\mathrm{m}*}(A, B) = \min_{c \in \mathcal{UN}(A,B)} 1 - coh^*(c)/coh^*(A, B),$$

$\alpha_2^{\mathrm{m}*}(A, B)$ may be interpreted as the truth degree of "each upper neighbor of $\langle A, B\rangle$ has a significantly smaller cohesion than $\langle A, B\rangle$", which is another possible reading of (BL2).

Ad (10): For the same reasons as in the case of (8), $\alpha_3^{\varnothing *}(A,B)$ may be interpreted as the truth degree of "on average, $\langle A,B\rangle$ has only lightly smaller cohesion than its lower neighbors", which is one possible meaning of (BL3).

Ad (11): For the same reasons as in the case of (10), $\alpha_3^{\mathrm{m}*}(A,B)$ may be interpreted as the truth degree of "$\langle A,B\rangle$ has only a slightly small than cohesion than each of its lower neighbors", which is another possible reading of (BL3).

The interpretation described in (7)–(11) is correct if, as was mentioned in the above remark, in the factions $\frac{coh^*(c_1)}{coh^*(c_2)}$ in (7)–(11), we have $coh^*(c_1) \le coh^*(c_2)$. This is the case of coh^{m}, as the following lemma shows.

Lemma 1. *If* $\langle A_1,B_1\rangle \le \langle A_2,B_2\rangle$ *then* $coh^{\mathrm{m}}(A_2,B_2) \le coh^{\mathrm{m}}(A_1,B_1)$.

Proof. Immediate to verify. □

However, for $coh^{\varnothing}$ such property no longer holds. Namely, the cohesion of an upper neighbor of a concept may be greater than that of the concept itself as the following example shows.

Example 1. Consider the formal context in Table 1. One may check that for the

	y_1	y_2	y_3	y_4	y_5
x_1	×	×	×		×
x_2	×	×		×	
x_3	×		×	×	×

Table 1. Formal context from Example 1.

formal concepts $\langle A_1,B_1\rangle = \{\{x_1,x_2\},\{y_1,y_2\}\}$ and $\langle A_2,B_2\rangle = \{\{x_1,x_2,x_3\},\{y_1\}\}$ we have $\langle A_2,B_2\rangle \in \mathcal{UN}(A_1,B_1)$ and yet:

$$coh^{\varnothing}(A_1,B_1) = \frac{sim(x_1,x_2)}{1} = \frac{2}{5} < \frac{\frac{2}{5}+\frac{3}{5}+\frac{2}{5}}{3} = \frac{sim(x_1,x_2)+sim(x_1,x_3)+sim(x_2,x_3)}{3} = coh^{\varnothing}(A_2,B_2),$$

for both $sim = sim_{\mathrm{SMC}}$ and $sim = sim_{\mathrm{J}}$.

We propose the following solution to this problem. Instead of considering $\mathcal{UN}(A,B)$, i.e. all the upper neighbors of $\langle A_1,B_1\rangle$, we consider only

$$\mathcal{UN}^{\le}(A,B) = \{c \in \mathcal{UN}(A,B) \mid coh^{\varnothing}(c) \le coh^{\varnothing}(A,B)\},$$

i.e. only the upper neighbors with a smaller cohesion in (8) and (9). In addition, it seems natural to disregard $\langle A,B\rangle$ as a candidate for a basic level concept if the number of "wrong upper neighbors" is relatively large, i.e. if $\frac{|\mathcal{UN}^{\le}(A,B)|}{|\mathcal{UN}(A,B)|} < \theta$

for some parameter θ. θ itself may be subject to experiments ($\theta = 1$ means that we require all the upper neighbors to have cohesion $\leq$ the cohesion of $\langle A, B\rangle$). Likewise, instead of considering $\mathcal{LN}(A, B)$, we consider only

$$\mathcal{LN}^{\geq}(A, B) = \{c \in \mathcal{LN}(A, B) \mid coh^{\varnothing}(c) \geq coh^{\varnothing}(A, B)\}$$

in (10) and (11), and a similar condition for the number of "wrong lower neighbors" given by θ. Therefore, we get the following formulas ($||\varphi||$ denoted the truth degree of condition φ):

$$\begin{aligned}
\alpha_1^*(A, B) &= coh^*(A, B), \\
\alpha_2^{\varnothing *}(A, B) &= [1 - \frac{\sum_{c \in \mathcal{UN}^{\leq}(A,B)} coh^*(c)/coh^*(A, B)}{|\mathcal{UN}^{\leq}(A, B)|}] \cdot || \frac{|\mathcal{UN}^{\leq}(A, B)|}{|\mathcal{UN}(A, B)|} \geq \theta||, \\
\alpha_2^{\mathrm{m}*}(A, B) &= [1 - \max_{c \in \mathcal{UN}^{\leq}(A,B)} coh^*(c)/coh^*(A, B)] \cdot || \frac{|\mathcal{UN}^{\leq}(A, B)|}{|\mathcal{UN}(A, B)|} \geq \theta||, \\
\alpha_3^{\varnothing *}(A, B) &= [\frac{\sum_{c \in \mathcal{LN}^{\geq}(A,B)} coh^*(A, B)/coh^*(c)}{|\mathcal{LN}^{\geq}(A, B)|}] \cdot || \frac{|\mathcal{LN}^{\geq}(A, B)|}{|\mathcal{LN}(A, B)|} \geq \theta||, \\
\alpha_3^{\mathrm{m}*}(A, B) &= [\min_{c \in \mathcal{LN}^{\geq}(A,B)} coh^*(A, B)/coh^*(c)] \cdot || \frac{|\mathcal{LN}^{\geq}(A, B)|}{|\mathcal{LN}(A, B)|} \geq \theta||.
\end{aligned}$$

Lemma 2. *$\alpha_2^{\varnothing *}(A, B) \geq \alpha_2^{\mathrm{m}*}(A, B)$ and $\alpha_3^{\varnothing *}(A, B) \geq \alpha_3^{\mathrm{m}*}(A, B)$.*

Proof. Immediate to verify.

Now, according to (6) every concept $\langle A, B\rangle$ gets assigned a degree $BL(A, B)$ to which $\langle A, B\rangle$ may be considered as a concept of a basic level. The concepts with high degrees are therefore considered as important ones. The degrees may be used to rank the concepts accordingly, i.e. to sort them from those with the highest basic level degrees to the lowest.

4 Experiments

We performed several experiments with the above method to identify basic level concepts. Our main aim was to see if the method is able to identify concepts that humans would naturally use as basic level concepts. Clearly, the subjectivity factor plays a significant role. We therefore selected datasets describing commonly known objects, for which most people would probably agree on the basic level concepts, or at least agree on whether a given concept can be regarded as a basic level concept.

We were not checking the results of our method for a given dataset against a psychological experiment involving a group of respondents telling their basic level concepts for the dataset. This important step, particularly from the psychology point of view, is left for future. Instead, we tried to see whether our method identifies basic level concepts in a reasonable way based on our intuition.

For every dataset $\langle X, Y, I\rangle$, we observed the basic level degrees of all concepts of the concept lattice $\mathcal{B}(X, Y, I)$. We report the results for the following combinations $BL_{\mathrm{s}}^{\mathrm{c,a}}(A, B)$: s is SMC or J and indicates whether sim_{SMC} or sim_{J} was used; c is $\varnothing$ or m and indicates whether $coh^{\varnothing}$ or coh^{m} was used; a is $\varnothing$ or m and indicates whether $\alpha_2^{\varnothing *}$ and $\alpha_3^{\varnothing *}$, or $\alpha_2^{\mathrm{m}*}$ and $\alpha_3^{\mathrm{m}*}$ was used. We report the results for $\theta = 1$. Note that we have:

Lemma 3. *$BL_{\mathrm{s}}^{\mathrm{c},\varnothing}(A, B) \geq BL_{\mathrm{s}}^{\mathrm{c,m}}(A, B)$ for any* s *and* c*, provided $\mathcal{C}$ is isotone.*

Proof. Directly from Lemma 2 and the fact that $\mathcal{C}$ in (6) is isotone.

4.1 Experiment 1

The dataset in Table 2 contains selected sports and their attributes. There are the following formal concepts in this dataset (for convenience, we list them in the form $c_i = \langle A, B\rangle$ where A and B are the extent and intent of c_i):

$c_1 = \langle\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}, \{\}\rangle$,
$c_2 = \langle\{1, 2, 4, 11, 13, 14, 15, 20\}, \{10\}\rangle$,
$c_3 = \langle\{3, 5, 6, 7, 8, 9, 10, 12, 16, 17, 18, 19\}, \{9\}\rangle$, $c_4 = \langle\{3, 4, 14, 16\}, \{8\}\rangle$,
$c_5 = \langle\{3, 16\}, \{8, 9\}\rangle$, $c_6 = \langle\{1, 2, 3, 4, 11, 13, 14, 15, 17, 19\}, \{5\}\rangle$,
$c_7 = \langle\{1, 2, 4, 11, 13, 14, 15\}, \{5, 10\}\rangle$, $c_8 = \langle\{3, 17, 19\}, \{5, 9\}\rangle$,
$c_9 = \langle\{3, 4, 14\}, \{5, 8\}\rangle$, $c_{10} = \langle\{4, 14\}, \{5, 8, 10\}\rangle$,
$c_{11} = \langle\{5, 6, 7, 8, 9, 10, 12, 16, 18, 20\}, \{4\}\rangle$,
$c_{12} = \langle\{5, 6, 7, 8, 9, 10, 12, 16, 18\}, \{4, 9\}\rangle$, $c_{13} = \langle\{5, 6, 7, 8, 9, 18\}, \{4, 7, 9\}\rangle$,
$c_{14} = \langle\{5, 6, 7, 8, 18\}, \{4, 6, 7, 9\}\rangle$, $c_{15} = \langle\{4, 16, 17, 18, 19, 20\}, \{3\}\rangle$,
$c_{16} = \langle\{4, 20\}, \{3, 10\}\rangle$, $c_{17} = \langle\{16, 17, 18, 19\}, \{3, 9\}\rangle$, $c_{18} = \langle\{4, 16\}, \{3, 8\}\rangle$,
$c_{19} = \langle\{4, 17, 19\}, \{3, 5\}\rangle$, $c_{20} = \langle\{17, 19\}, \{3, 5, 9\}\rangle$, $c_{21} = \langle\{16, 18, 20\}, \{3, 4\}\rangle$,
$c_{22} = \langle\{20\}, \{3, 4, 10\}\rangle$, $c_{23} = \langle\{16, 18\}, \{3, 4, 9\}\rangle$, $c_{24} = \langle\{16\}, \{3, 4, 8, 9\}\rangle$,
$c_{25} = \langle\{18\}, \{3, 4, 6, 7, 9\}\rangle$, $c_{26} = \langle\{9, 10, 11, 12, 13, 14, 15\}, \{2\}\rangle$,
$c_{27} = \langle\{11, 13, 14, 15\}, \{2, 5, 10\}\rangle$, $c_{28} = \langle\{14\}, \{2, 5, 8, 10\}\rangle$,
$c_{29} = \langle\{9, 10, 12\}, \{2, 4, 9\}\rangle$, $c_{30} = \langle\{9\}, \{2, 4, 7, 9\}\rangle$,
$c_{31} = \langle\{1, 2, 3, 4, 5, 6, 7, 8\}, \{1\}\rangle$, $c_{32} = \langle\{3, 5, 6, 7, 8\}, \{1, 9\}\rangle$,
$c_{33} = \langle\{1, 2, 3, 4\}, \{1, 5\}\rangle$, $c_{34} = \langle\{1, 2, 4\}, \{1, 5, 10\}\rangle$, $c_{35} = \langle\{3, 4\}, \{1, 5, 8\}\rangle$,
$c_{36} = \langle\{3\}, \{1, 5, 8, 9\}\rangle$, $c_{37} = \langle\{5, 6, 7, 8\}, \{1, 4, 6, 7, 9\}\rangle$,
$c_{38} = \langle\{4\}, \{1, 3, 5, 8, 10\}\rangle$, $c_{39} = \langle\{\}, \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}\rangle$

Table 3 contains concepts c_1–c_{39} and their basic level degrees. The corresponding concept lattice is depicted using a reduced labeling in Fig. 1.

Note first that in accordance with Lemma 3, the basic level degrees in column $BL_{\mathrm{SMC}}^{\varnothing\varnothing}$ are greater than or equal to those in column $BL_{\mathrm{SMC}}^{\varnothing\mathrm{m}}$; and that the same is true when comparing columns $BL_{\mathrm{SMC}}^{\mathrm{m}\varnothing}$ to $BL_{\mathrm{SMC}}^{\mathrm{mm}}$, $BL_{\mathrm{J}}^{\varnothing\varnothing}$ to $BL_{\mathrm{J}}^{\varnothing\mathrm{m}}$, and $BL_{\mathrm{J}}^{\mathrm{m}\varnothing}$ to $BL_{\mathrm{J}}^{\mathrm{mm}}$.

Second, note that it seems not to matter very much whether $\alpha_2^{\varnothing *}$ and $\alpha_3^{\varnothing *}$, or $\alpha_2^{\mathrm{m}*}$ and $\alpha_3^{\mathrm{m}*}$ is used. This observation needs to be explored further in future. On the other hand, it matters significantly whether $coh^{\varnothing}$ or coh^{m} is used. According to our intuition and the results of this and other experiments we performed, we

		on land	on ice	in water	collective sport	individual sport	using ball	needs opponent	multiple disciplines	points	time
		1	2	3	4	5	6	7	8	9	10
Run	1	×				×					×
Orienteering	2	×				×					×
Gymnastics	3	×				×			×	×	
Triathlon	4	×		×		×			×		×
Football	5	×			×		×	×		×	
Inline Hockey	6	×			×		×	×		×	
Tennis	7	×			×		×	×		×	
Baseball	8	×			×		×	×		×	
Ice Hockey	9		×		×			×		×	
Curling	10		×		×					×	
Cross-country Skiing	11		×			×					×
Synchronized Skating	12		×		×					×	
Alpine Skiing	13		×			×					×
Biathlon	14		×			×			×		×
Speed Skating	15		×			×					×
Synchronized Swimming	16			×	×				×	×	
Diving	17			×		×				×	
Water Polo	18			×	×		×	×		×	
Underwater Diving	19			×		×				×	
Rowing	20			×	×						×

Table 2. Sports and their attributes.

hypothesize that $coh^{\varnothing}$ is better to use than coh^{m}. Again, a more detailed study is needed to support this claim.

Third, and most importantly for the purpose of our paper, let us consider the concepts that have been indicated as basic level concepts by the method. Due to lack of space, we consider the selection by $BL^{\varnothing\varnothing}_{\mathrm{SMC}}$ only. The concepts with a non-zero degree, depicted by square nodes in Fig. 1, sorted in a descending way are c_{29} (which can be verbally described as "winter collective sports"; encompassing Ice Hockey, Curling, Synchronized Skating), c_{27} ("individual winter sports"; Cross-country Skiing, Alpine Skiing, Biathlon, Speed Skating), c_{32} ("land sports evaluated by points"), c_{26} ("winter sports"), c_{13} ("collective sports with opponent"), c_8 ("individual sports"), c_{14} ("ball games"), c_{34} ("land sports evaluated by time"), c_{19} ("individual water sports"), c_{31} ("land sports"). Arguably, all of them are likely to be considered natural, basic level concepts. On

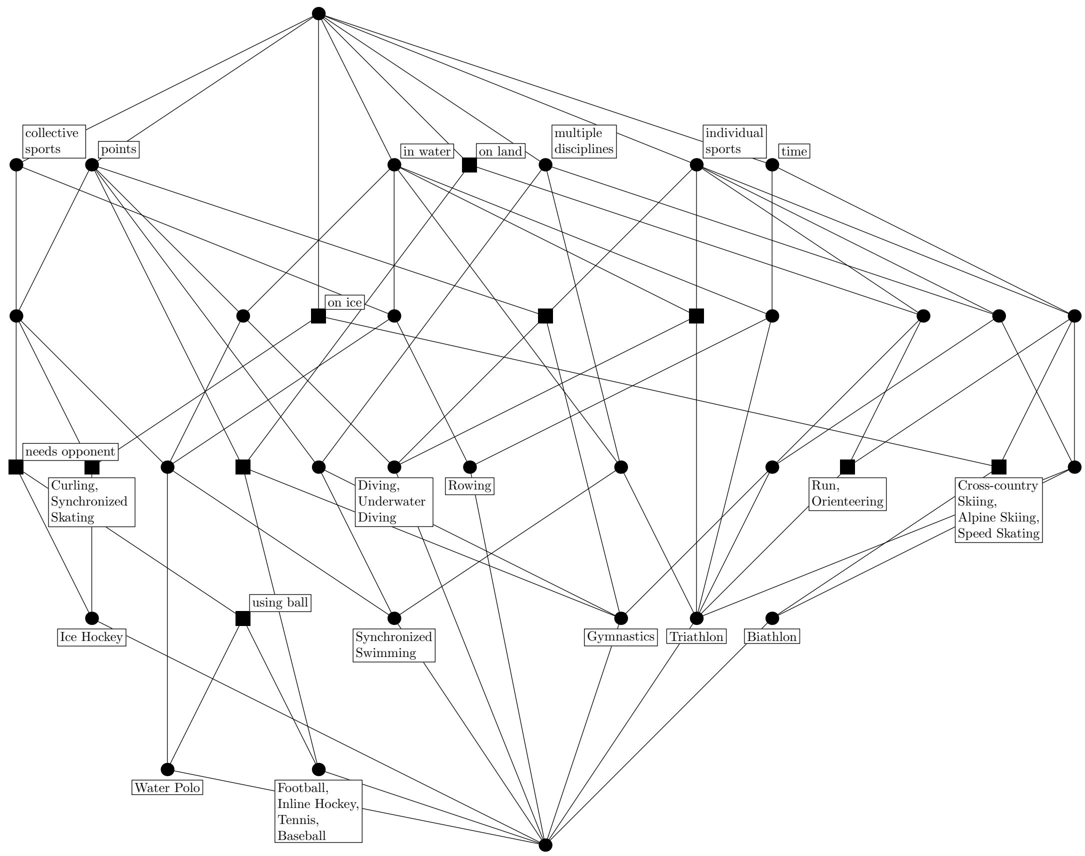

Fig. 1. Concept lattice of Table 2.

the other hand, among the concepts not selected for basic level are, for example, c_{35} ("individual land sports with multiple disciplines", encompassing Gymnastics and Triathlon), c_{30} ("collective winter sports with opponent evaluated by points" consisting of Ice Hockey), c_{28} ("individual winter sports with multiple disciplines evaluated by time" consisting of Biathlon), or c_{18} ("sports performed in water with multiple disciplines" encompassing Triathlon and Synchronized Swimming). These concepts are not likely to be regarded as basic level concepts. From this and other experiments we conclude that the method we present in this paper tends to select natural concepts likely to be considered basic level concepts and not to select clear non-basic level concepts. For the "borderline cases", however, it happens that seemingly natural concepts are not selected and, vice versa, that the selected concepts do not seem likely to be considered basic level concepts. It has to be noted that this study presents an initial study in the presented problem. As such, we consider the method promising and giving good results already at this stage.

An important observation, that we made in several experiments, worth noting here is that which concepts are considered as basic level concepts very much depends on the dataset and the selected attributes in particular. Typically, a human expert tends to take into account other information (not only the attributes present in the dataset) to assess which concepts are the basic level concepts. This makes it difficult to objectively assess the quality of a particular basic level function. Rather than instructing the human expert "You must use only the attributes and objects in the dataset to tell which concepts are basic level concepts for our data", which the expert is likely to inadvertently disobey anyway, we learned that the dataset must be balanced in that it must contain the main relevant attributes people would naturally take into account when telling the basic level concepts. From this point of view, it is certainly desirable to do more experiments with the sport dataset and its variants, such as the one that would distinguish attribute "points" as to whether points are assigned by jury or whether this means that points are scored such as in Ice Hockey, which indeed impacts the basic level.

4.2 Experiment 2

The dataset in Table 4 contains selected animals and their attributes. Table 5 contains the concepts of this dataset and their basic level degrees. Due to lack of space, we leave the interpretation of these results to the reader.

5 Conclusions and Future Research

We proposed a method that utilizes a psychological phenomenon of basic level of concepts to select possibly important, natural concepts from a concept lattice and presented first results and experience obtained from experiments. The future research will focus on the following issues:

- Psychological experiments. First, to help assess the quality of the functions for the basic level degrees, aiming to benefit the process of selecting important concepts from the concept lattice. Second, to help better understand the phenomenon of the basic level, thus aiming to benefit the psychology of concepts itself. Our experimental work opens several questions for psychological research such as whether the basic level may contain comparable concepts or whether and in what sense the collection of basic level concepts needs to be exhaustive.
- Comparison with other techniques to select important formal concepts, in particular with the stability index [14, 15]. A more detailed study of the the mutual relationship of the various basic level degree functions, utilizing statistical analyses.
- Utilizing further results of the studies of the basic level in the psychology of concepts, in particular utilizing those from which quantitative criteria for the basic level can be obtained[19, p. 213].

– Comparing the idea of basic level concepts with the heuristics known from cluster analysis [9].
– A theoretical study of the issues pertaining to basic level, involving existing work on similarity in concept lattices such as [17].
– Design of efficient algorithms to compute basic level concepts.

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intent of concept $\langle A, B\rangle$										basic level degree of $\langle A, B\rangle$							
on land	on ice	in water	collective sport	individual sport	using ball	needs opponent	multiple disciplines	points	time	$BL^{\emptyset\emptyset}_{\mathrm{SMC}}$	$BL^{\emptyset\mathrm{m}}_{\mathrm{SMC}}$	$BL^{\mathrm{m}\emptyset}_{\mathrm{SMC}}$	$BL^{\mathrm{mm}}_{\mathrm{SMC}}$	$BL^{\emptyset\emptyset}_{\mathrm{J}}$	$BL^{\emptyset\mathrm{m}}_{\mathrm{J}}$	$BL^{\mathrm{m}\emptyset}_{\mathrm{J}}$	$BL^{\mathrm{mm}}_{\mathrm{J}}$
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0.33	0.33	0	0	0.08	0.08
0	0	0	0	0	0	0	0	1	0	0	0	0.11	0.09	0	0	0.05	0.04
0	0	0	0	0	0	0	1	0	0	0	0	0.21	0.20	0	0	0.06	0.06
0	0	0	0	0	0	0	1	1	0	0	0	0.15	0.12	0	0	0.07	0.06
0	0	0	0	1	0	0	0	0	0	0	0	0.31	0.29	0.12	0.11	0.08	0.07
0	0	0	0	1	0	0	0	0	1	0	0	0.08	0.07	0	0	0.09	0.07
0	0	0	0	1	0	0	0	1	0	0.10	0.07	0.21	0.14	0.10	0.10	0.10	0.09
0	0	0	0	1	0	0	1	0	0	0	0	0.13	0.09	0.05	0.01	0.12	0.11
0	0	0	0	1	0	0	1	0	1	0	0	0.07	0.07	0	0	0.08	0.08
0	0	0	1	0	0	0	0	0	0	0	0	0.22	0.20	0	0	0.07	0.06
0	0	0	1	0	0	0	0	1	0	0	0	0.10	0.06	0	0	0.08	0.05
0	0	0	1	0	0	1	0	1	0	0.11	0.10	0.16	0.14	0.16	0.15	0.13	0.11
0	0	0	1	0	1	1	0	1	0	0.07	0.07	0.08	0.08	0.11	0.11	0.11	0.11
0	0	1	0	0	0	0	0	0	0	0	0	0.03	0.03	0	0	0.04	0.04
0	0	1	0	0	0	0	0	0	1	0	0	0.15	0.06	0	0	0.06	0.06
0	0	1	0	0	0	0	0	1	0	0	0	0.26	0.18	0.08	0.04	0.11	0.07
0	0	1	0	0	0	0	1	0	0	0	0	0.10	0.05	0	0	0.05	0.04
0	0	1	0	1	0	0	0	0	0	0.05	0.02	0.15	0.06	0.09	0.07	0.06	0.06
0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0	0	0	0.22	0.12	0	0	0.10	0.06
0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	1	0	0	0	0.09	0.07	0	0	0.09	0.08
0	0	1	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	0	1	1	0	1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0.14	0.14	0.13	0.13	0.11	0.10	0.03	0.03
0	1	0	0	1	0	0	0	0	1	0.18	0.13	0.36	0.27	0.29	0.23	0.38	0.31
0	1	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	1	0	0.20	0.19	0.41	0.36	0.27	0.24	0.40	0.35
0	1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0.05	0.05	0.03	0.03	0.11	0.10	0.04	0.03
1	0	0	0	0	0	0	0	1	0	0.15	0.13	0.13	0.10	0.21	0.20	0.05	0.05
1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0.19	0.16
1	0	0	0	1	0	0	0	0	1	0.06	0.05	0.12	0.08	0.10	0.09	0.14	0.12
1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0.07	0.05
1	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0
1	0	0	1	0	1	1	0	1	0	0	0	0	0	0	0	0	0
1	0	1	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0

Table 3. Basic level degrees of the concepts of Table 2.

	gives milk	gives meat	gives fur	gives eggs	warmblooded	coldblooded	walks on 2 legs	walks on 4 legs	lives in water	lives on land	flies	usually has name	omnivore	carnivore	herbivore	used to pull
German Shepherd					×			×		×		×		×		
Labrador Retrieval					×			×		×		×		×		
European Shorthair Cat					×			×		×		×		×		
Persian Cat					×			×		×		×		×		
Sheep	×	×	×		×			×		×					×	
Ouessan Sheep	×	×	×		×			×		×					×	
Domestic Goat	×	×			×			×		×					×	
Mountain Goat		×			×			×		×					×	
Chicken		×		×	×		×			×	×		×			
Czech Gold Chicken		×		×	×		×			×	×		×			
Pig		×			×			×		×			×			
Indochinese Warty Pig		×			×			×		×			×			
Domestic Horse		×			×			×		×		×			×	×
Pony		×			×			×		×		×			×	×
Donkey	×	×			×			×		×		×			×	×
Rabbit		×			×			×		×					×	
Coypu		×			×			×	×	×					×	
Kosovo Rooster		×		×	×		×			×	×		×			
Domestic Goose		×	×	×	×		×		×	×	×				×	
Lesser White-fronted Goose	×	×	×	×		×		×	×	×				×		
Wild Mallard Duck		×	×		×		×		×	×	×		×			
Domestic Duck		×	×	×	×		×		×	×	×		×			
Domestic Pigeon		×			×		×			×	×				×	
Common Ostrich		×	×	×	×		×			×			×			
Domestic Turkey		×		×	×		×			×					×	
Budgerigar					×		×			×	×	×			×	
Carp		×				×			×				×			
Trout		×				×			×					×		

Table 4. Animals and their attributes.

intent of concept $\langle A, B\rangle$																basic level degree of $\langle A, B\rangle$							
gives milk	gives meat	gives fur	gives eggs	warmblooded	coldblooded	walks on 2 legs	walks on 4 legs	lives in water	lives on land	flies	usually has name	omnivore	carnivore	herbivore	used to pull	$BL^{\emptyset\emptyset}_{\mathrm{SMC}}$	$BL^{\emptyset\mathrm{m}}_{\mathrm{SMC}}$	$BL^{\mathrm{m}\emptyset}_{\mathrm{SMC}}$	$BL^{\mathrm{mm}}_{\mathrm{SMC}}$	$BL^{\emptyset\emptyset}_{\mathrm{J}}$	$BL^{\emptyset\mathrm{m}}_{\mathrm{J}}$	$BL^{\mathrm{m}\emptyset}_{\mathrm{J}}$	$BL^{\mathrm{mm}}_{\mathrm{J}}$
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	1	0	0	1	0	1	0	0	0	0	1	0	0.03	0.03	0.05	0.05	0.06	0.06	0.07	0.07
1	1	1	0	1	0	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
1	1	0	0	1	0	0	1	0	1	0	1	0	0	1	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.04	0.04
0	1	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0.02	0.02	0.14	0.13	0.07	0.07	0.08	0.07
0	1	1	1	1	0	1	0	0	1	0	0	0	0	0	0	0.01	0	0	0	0.03	0.01	0	0
0	1	1	1	1	0	1	0	1	1	1	0	0	0	0	0	0.05	0.03	0.09	0.05	0.07	0.04	0.13	0.08
0	1	1	1	1	0	1	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	0	1	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	1	1	1	1	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0.08	0.02
0	1	1	0	1	0	1	0	0	1	0	0	0	0	0	0	0.07	0.03	0	0	0.11	0.06	0	0
0	1	1	0	1	0	1	0	1	1	1	0	0	0	0	0	0.06	0.03	0.13	0.05	0.10	0.06	0.18	0.08
0	1	1	0	1	0	1	0	1	1	1	0	1	0	0	0	0.09	0.04	0.18	0.12	0.14	0.06	0.27	0.17
0	1	1	0	1	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	1	1	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0.10	0.08	0.03	0.02	0.06	0.06
0	1	0	1	1	0	1	0	0	1	0	0	0	0	0	0	0.02	0.01	0.05	0.05	0.03	0.03	0.08	0.07
0	1	0	1	1	0	1	0	0	1	1	0	0	0	0	0	0.02	0.02	0.05	0.05	0.05	0.04	0.08	0.08
0	1	0	1	1	0	1	0	0	1	1	0	1	0	0	0	0.04	0.02	0	0	0.07	0.05	0.08	0.02
0	1	0	1	1	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0.16	0.08
0	1	0	1	1	0	1	0	0	1	0	0	0	0	1	0	0.03	0.02	0	0	0.04	0.04	0	0
0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0.03	0.02	0	0	0.04	0.04	0.05	0.02
0	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0.07	0.02	0.16	0.05	0.10	0.04	0.10	0.06
0	1	0	0	1	0	1	0	0	1	1	0	0	0	0	0	0.02	0.01	0.08	0.05	0.04	0.03	0.09	0.07
0	1	0	0	1	0	1	0	0	1	1	0	1	0	0	0	0.04	0.01	0	0	0.06	0.03	0	0
0	1	0	0	1	0	1	0	0	1	1	0	0	0	1	0	0.04	0.02	0	0	0.05	0.04	0	0
0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0.05	0.03	0	0	0.08	0.04	0	0
0	1	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0.08	0.04	0.18	0.04	0.10	0.06	0.11	0.05
0	1	0	0	1	0	0	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	0	1	0	1	0	1	0	0	1	1	0.11	0.09	0.23	0.18	0.20	0.18	0.36	0.28
0	1	0	0	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	1	0	0	1	0	1	0	0	0	0	1	0	0.06	0.02	0.15	0.05	0.07	0.04	0.12	0.06
0	1	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0.10	0.07	0.14	0.09	0.17	0.12	0.10	0.08
0	1	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0.01	0	0.13	0.09	0.03	0	0.08	0.07
0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0.07	0.04	0.13	0.04	0.09	0.06	0.09	0.07
0	1	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0.04	0.02	0.03	0
0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0.19	0.19	0.38	0.38	0.09	0.09	0.25	0.25
0	1	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0.09	0.06	0	0	0.04	0.03
0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0.03	0.02
0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0.09	0.07
0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0.02	0.02	0	0	0.04	0.03	0.09	0.07
0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0.11	0.11	0	0	0.18	0.17	0.10	0.08
0	0	0	0	1	0	1	0	0	1	1	0	0	0	0	0	0.01	0.01	0	0	0.02	0.02	0.05	0.05
0	0	0	0	1	0	1	0	0	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	1	0	0	1	1	0	0	0	1	0	0	0	0	0	0.01	0.01	0	0
0	0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0.05	0.01	0	0
0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0.11	0.11	0.28	0.28	0.11	0.10	0.13	0.13
0	0	0	0	1	0	0	1	0	1	0	1	0	0	0	0	0.04	0.03	0.04	0.04	0.07	0.05	0.04	0.03
0	0	0	0	1	0	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0.12	0.11	0	0	0.14	0.14	0.19	0.18
0	0	0	0	1	0	0	0	0	1	0	1	0	0	1	0	0.03	0.01	0	0	0.06	0.04	0.03	0.01
0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0.07	0.07	0	0	0.10	0.09	0.10	0.08
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0.14	0.14	0.14	0.14	0.15	0.15	0.02	0.02

Table 5. Basic level degrees of the concepts of Table 4.