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Fuzzifier's temptation

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Abstract

I present my views of fuzzification from the viewpoint of research practice in fuzzy logic. My particular aim is to describe a significant phenomenon—which I call fuzzifier's temptation—and discuss its ramifications at the present stage of development of fuzzy logic.

Keywords: Fuzzy logic; Fuzzification

1. Fuzzy logic and its development

Although the term "fuzzy logic" has multiple connotations, its common ground is the view that truth is a matter of degree [8,9]. That is, fuzzy logic rejects a fundamental principle of classical logic—the principle of bivalence—, according to which every proposition has only two possible truth values, true and false. Fuzzy logic does not abandon these two classical truth values but rather allows for additional ones. These truth values are interpreted as degrees of truth and are considered as intermediate between false and true.

Since classical logic represents a long-standing formal foundation for human reasoning and forms a pillar of modern science, the shift from classical to fuzzy logic opens a fundamentally new perspective on virtually all phenomena in philosophy, logic, mathematics, science, as well as applied fields. In this view, it is apparent that a proper development of fuzzy logic and its applications is a rather complex process.

A key aspect of this development derives from the profound nature of the shift from classical to fuzzy logic. Namely, the shift concerns the very basic notions in logic, and has therefore a deep significance for methodological matters.[1] As all branches of modern science rely on classical logic, a proper development of fuzzy logic and the theories based on it requires a thorough reconsideration of fundamentals. Moreover, within the new setting of fuzzy logic, new concepts and ideas arise that do not exist or are degenerate in the classical setting. Exploration of these new prospects is arguably the most challenging and most important part of the development of fuzzy logic.

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[1] Some consider fuzzy logic a new paradigm in the sense of Thomas Kuhn.

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0165-0114/

From a mathematical viewpoint, the setting of fuzzy logic is naturally regarded as generalizing the setting of classical logic[2]: Many concepts, theories, and methods developed within the setting of fuzzy logic generalize the classical ones, which appear their particular cases implied by the constraints imposed by the laws of classical logic. The central theme of this paper—fuzzification and what I call the fuzzifier's temptation—is inherently associated precisely with this particular facet of the development of fuzzy logic.

2. Fuzzification

The so-called *fuzzification* played a significant role in the development of fuzzy logic since its inception [1–6]. In its basic meaning, fuzzification is synonymous with a generalization from the viewpoint of fuzzy logic. One may thus fuzzify the classical notion of relation to obtain a notion of fuzzy relation, fuzzify classical theorems, classical theories, or even whole areas of mathematics and other fields which have been developed within the framework of classical logic.

As an example, consider the classical notion of a binary relation. For the purpose of fuzzification, a binary relation between sets U and V may conveniently be identified with a function $R : U \times V \to \{0, 1\}$ for which $R(u, v) = 1$ if $u \in U$ and $v \in V$ are related and $R(u, v) = 0$ if u and v are not related. The corresponding fuzzified notion, that of a binary fuzzy relation, is obtained in a straightforward manner in this case, namely by replacing the set $\{0, 1\}$ of classical truth values by the interval $[0, 1]$ of truth degrees, defining thus a binary fuzzy relation as a function $R : U \times V \to [0, 1]$. One may then fuzzify various properties of relations to obtain the corresponding properties of fuzzy relations. For instance, transitivity of a fuzzy relation R requires $R(u, v) \otimes R(v, w) \leq R(u, w)$ for any u, v, and w, where $\otimes$ is an appropriate truth function of conjunction, generalizing indeed transitivity of classical relations, which demands that if u is related to v and v is related to w, then u is related to w. Fuzzification may get rather non-trivial and technically involved. Nevertheless, a sound generalization from the classical setting to the setting involving degrees of truth remains the principal issue.

3. Fuzzifier's temptation

Fuzzification represents a useful and indispensable method in the development of fuzzy logic as well as the various areas of human inquiry based on fuzzy logic. Yet, it connects to a phenomenon which is detrimental to this development and which is apparent particularly when observing the research practice in fuzzy logic. I call this phenomenon the *fuzzifier's temptation*. Put briefly, the fuzzifier's temptation leads to a production of insubstantial fuzzifications.[3] Such production and the related practice are unhealthy for the development of fuzzy logic, and harm the field in other respects as well. The aim of this paper is to describe this phenomenon, present my view of it, and discuss its ramifications for the development of fuzzy logic.[4]

3.1. What is fuzzifier's temptation?

To a great extent, fuzzifier's temptation originates from the following, seemingly coherent prospect regarding the development of fuzzy logic and the areas of human inquiry based on it: Classical logic, classical mathematics, as well as other areas based on classical logic have developed to the current state with their established concepts, theories, and methods through a long, demanding process of diligently making new discoveries, which was paved by trial and error. This state has proved useful by numerous applications to real world problems which utilize the developed concepts, theories, and methods. Thus, in particular, classical mathematics developed to its current stage with its

[2] There are several ways to consider what it means exactly that the setting of fuzzy logic generalizes the setting of classical logic; they are basically due to the fact that fuzzy logic admits intermediate truth degrees and that it contains logical connectives that behave classically when applied to classical truth values.

[3] I am not primarily interested in examples of such fuzzifications in this paper, and refrain from referring to any papers presenting such fuzzifications.

[4] It is to be noted that what I describe below has to varying extents been informally discussed on various occasions by fuzzy logic researchers. My aim in this regard is to provide a coherent account of the problems involved. I first used the term "fuzzifier's temptation" in [1]; this book contains initial thoughts on the matters discussed here.

organization into algebra, geometry, topology, calculus, and other branches, through a natural process and for good reason. Since fuzzy logic generalizes classical logic to a setting that is more realistic as regards human reasoning, namely by allowing intermediate truth degrees, it is desirable—the reasoning continues—to systematically fuzzify established classical concepts, theories, and methods, and hence eventually fuzzify whole areas of human inquiry. In particular, it is desirable to fuzzify classical mathematics, and to develop fuzzy algebra, fuzzy geometry, fuzzy topology, and so on. The mathematics developed this way shall have a greater scope of applicability, especially when encountering problems which involve non-bivalent concepts, i.e. concepts with unsharp, fuzzy boundaries. Hence—the reasoning concludes—, a systematic fuzzification of classical mathematics and other areas of human inquiry that are based on classical logic represents a reasonable research program.

This program suffers from several fundamental problems to be discussed next. Still, browsing the literature on fuzzy logic it becomes apparent that many publications may be regarded as pursuing this program. Appearance of such publications may be observed ever since the early years of fuzzy logic. With progress of time, their frequency is increasing, and became quite considerable recently to the extent which is disturbing in my view.[5]

I shall call the temptation to pursue the research program described above the *fuzzifier's temptation*. The program itself shall be called the blind fuzzifier's program.[6]

Let me stress again that fuzzification itself is an important and useful method in the development of fuzzy logic. The problem I address consists in the pursuit—deliberate or unconscious—of the blind fuzzifier's program.[7]

3.2. Principal problem with fuzzifier's temptation

The principal problem consists in the idea, which sits at the program's core, namely that by fuzzification of useful classical concepts, theories, and methods one obtains useful concepts, theories, and methods again. Such assumption is not warranted. Naturally, the concepts, theories, and methods obtained by fuzzification prove useful in many cases. The point, however, is that—put bluntly—usefulness of a classical concept does not imply usefulness of its fuzzified version.

Namely, for a concept, a theory, or a method to eventually prove useful, it needs to evolve—via a complex process paved by trial and error—to a form that is mathematically correct, elegant and fitting well together with the already established concepts and theories, and suitable for solving substantial problems. While the first component, i.e. mathematical correctness, is in a sense trivial, the other two components are crucial. Elegance and fitting the established concepts and theories is basically achieved through a careful theoretical inspection of the new, emerging concepts and theories, i.e. through examination of their properties and their interactions with the existing, already established ones. This theoretical inspection may be regarded as a mathematical experimentation with the emerging concepts and theories. Indispensable and complementing the mathematical experimentation is the third component, namely, suitability to solve important problems. Such problems typically come from real world applications, but may also come from theoretical explorations.[8] It is the connection of the new concept, new theory, or new method to real world problems and the ability to solve such problems that ultimately justifies the introduction and further examination of the new concept, theory, or method.

[5] Even though I deliberately refrain from referring to such publications, I include for the purpose of illustration the following example, which concerns fuzzy subalgebras. This topic goes back to Rosenfeld's paper [7]. Rosenfeld introduced the concept of a fuzzy subgroupoid of a classical groupoid $\langle G, \cdot \rangle$ as a fuzzy set $A : G \to [0, 1]$ in G satisfying $A(g) \otimes A(h) \leq A(g \cdot h)$ for any $g, h \in G$, with $\otimes$ being the minimum conjunction, introduced the concept of a fuzzy subgroup of a classical group, and presented their basic properties. Except for presenting a natural possibility to apply the then new concept of a fuzzy set in algebra, Rosenfeld had not introduced any practical motivations for his notion. Nevertheless, his concept, which is basically the concept of closedness of a fuzzy set under an operation, proved interesting since several examples of fuzzy subgroups have been found; see e.g. [1]. Later on, however, Rosenfeld's idea has been used by many authors to publish papers, with no motivation and justification whatsoever except for referring to Rosenfeld's work, introducing fuzzy subalgebras of various classical algebraic structures. Given Rosenfeld's work, the concepts as well as the results in those papers are rather routine. Such results represent what I call insubstantial fuzzifications. Such fuzzifications are frequently appearing in algebra, but are common in virtually any field.

[6] To be sure, the program as described above has not been explicitly formulated in the literature. I nevertheless claim that my description of the program captures well the spirit of numerous existing research works—those that may be regarded as pursuing the program.

[7] Note at this point that I believe that a deliberate pursuit of the program tends to lead to an unconscious pursuit by researchers who join the field of fuzzy logic at a later stage. This harmful aspect is briefly touched upon below.

[8] For instance, a new concept may be useful for it enables us to solve a long-standing theoretical problem.

In this regard, the natural process described above is grossly distorted by the assumption of the blind fuzzifier's program that the usefulness of fuzzified concepts, theories, and methods is guaranteed by the established usefulness of their classical counterparts. As a consequence, those pursuing the blind fuzzifier's program unduly focus on formal aspects of fuzzification (the first component above), while neglecting other aspects for obtaining a useful concept, theory, or method (the second and third components above). In other words, fuzzifier's temptation leads to a shallow exploration of the involved concepts, to a considerable detachment from applications, and to unhealthy restriction to formalistic treatment of the explored subjects.

3.3. Amplifiers of fuzzifier's temptation

Prior to describing further problems with and ramifications of the fuzzifier's temptation, let me describe three factors that amplify its effect. The first factor may be termed a *low entry cost,* by which I mean the following. The basic concepts of fuzzy logic, such as the concept of a truth degree, the concept of a fuzzy set or a fuzzy relation, are mathematically straightforward and intuitive. Given a classical concept, it hence does not take much insight and perspicacity to define a fuzzification of the given concept. As a matter of fact, natural fuzzifications of simple concepts often result by a straightforward replacement of sets by fuzzy sets, relations by fuzzy relations, and properties of sets and relations by suitable properties of fuzzy sets and fuzzy relations. Even though a well-thought fuzzification of a given concept may require non-trivial considerations, this fact may not prevent an eager researcher from putting quickly forward *some* fuzzification and making it the topic of a research paper. The point here is that the cost of entry to the field of fuzzy logic is low with respect to the basic concepts, or at least their simple versions, compared to the costs of entry to other fields such as stochastic differential equations or mathematical models of quantum computation. For this reason, the low entry cost apparently amplifies the effect of fuzzifier's temptation.

The second factor having an amplifying effect is related to the well-known growth in the number of research publications. There are several grounds for this growth, among which is the "publish or perish" rule imposed on researchers by universities and funding agencies. The resulting pressure to publish makes researchers look for easy ways to produce research outputs. Systematically fuzzifying whole areas of classical theories is one such way.

The growth in the number of research publications is magnified by the accompanying decline in quality of peer review resulting from the lack of competent reviewers. This situation is manifested by the varying quality of research published especially in non-established journals. While this effect may be observed in virtually all areas of research, the decline of review quality particularly encourages publication of easy results such as insubstantial fuzzifications of questionable value.

3.4. Further problems and ramifications of fuzzifier's temptation

While the intention to fuzzify a given classical concept might seem to leave the fuzzifier with a relatively straightforward prospect of what to do, this in reality is rarely the case. The problem is that, more often than not, a given concept allows for multiple fuzzifications, each of which may be carried out correctly from a purely mathematical point of view. Naturally, a choice of a particular fuzzification to be pursued would result from considering the particular application for which the fuzzified concept is intended. The above mentioned detachment from applications, characteristic of the pursuit of the blind fuzzifier's program, results in having a limited or no ground for making such a choice.

For example, what does it mean to fuzzify the classical concept of a vector space, when one has no particular application in mind? Shall the set of vectors be replaced by a fuzzy set of vectors in the new concept of a fuzzy vector space? Or shall rather the set of scalars be made fuzzy, or the operation of adding vectors be replaced by some kind of a fuzzified operation? With no particular application in mind, such questions cannot be answered.

When detached from applications, the only option left for the pursuit of the blind fuzzifier's program is to consider all possibilities. That is to say, to consider all the possible fuzzifications of a given classical concept as equally meaningful and potentially interesting to explore. This would represent a purely formalistic way to do mathematical research. A solid mathematical practice, however, is not a formalistic enterprise. It must involve the components described in section 3.2.

The pursuit of the blind fuzzifier's program also leads to an undesirable restriction of the scope in exploring fuzzy logic. On the one hand, it is true that many mathematical concepts instrumental for fuzzy logic, starting with the very concept of a fuzzy set, result by fuzzification of classical concepts. However, several concepts essential for fuzzy logic do not arise this way. Exploration of such concepts is in fact essential for a successful development of fuzzy logic. A well-known example is averaging of truth degrees: While this is a useful concept in fuzzy logic, there is no classical counterpart to this concept because truth values cannot be averaged in classical logic. As is well known, there are numerous concepts of this sort. Some, such as the concept of an a-cut of a fuzzy set have their classical counterparts but do not naturally arise by fuzzification either: The classical counterpart of the concept of a-cut is degenerate and thus in a sense non-existent in the classical setting. While the discovery of concepts that do not exist in the classical setting but appear significant in the setting of fuzzy logic may arguably be considered of paramount importance, fuzzifier's temptation distracts from such a discovery.

4. Personal prospect

The literature on fuzzy logic contains a considerable amount of papers with questionable theoretical and practical value. It may be argued that this condition applies to many research fields and that it mainly results from the publication boom in academia we have witnessed for quite some time. However, a great deal of these questionable papers in the area of fuzzy logic may specifically be described as works concerned with insubstantial fuzzifications of various subjects. While this view is, by and large, shared within the fuzzy logic community, an analysis of the situation I characterize is missing in the literature.

I gradually became convinced that the problem of publishing research concerned with insubstantial fuzzifications should be addressed. This conviction grew stronger particularly during my tenure as area editor for algebra in the journal of *Fuzzy Sets and Systems*. I eventually came to the conclusion that the situation with insubstantial fuzzifications needed to be expounded and that its basic analysis needed to be worked out. The main aim in this paper is to provide such an analysis and, in particular, provide an explicit description of a phenomenon, termed fuzzifier's temptation and presented in section 3, which in my view is causing the present situation.

4.1. Should one be concerned?

The appearance of papers with questionable quality represents a worrisome problem for any field. It may be argued that to a certain extent, papers of questionable quality appear in virtually every research area. The amount of papers concerned with insubstantial fuzzifications, however, is particularly concerning in my view. Such papers earn fuzzy logic a particularly bad press. To an outside observer the situation suggests the impression that research in fuzzy logic largely consists in fuzzifying classical theories and methods, which is moreover being carried out without proper scientific discipline, in particular with a lack of solid motivations and with insufficient experimental work. Such impression is especially harmful when affecting students and young researchers who consider pursuing fuzzy logic as their topic. For these reasons, the ramifications of fuzzifier's temptation are severe and are worth serious consideration.

4.2. Possible standpoint

While the view presented in the preceding sections hopefully provides a better understanding of the present situation, it begs the question of a proper stance on fuzzifier's temptation to be taken by reviewers and editors. In this regard, I suggest the following theses, which in my view form a coherent set of propositions in the current stage in the development of fuzzy logic as regards fuzzification from a research practice perspective.

- Fuzzification represents a well-established method.
  Fuzzification has proven useful in fuzzy logic. Many concepts and methods which resulted by fuzzification of their classical counterparts are considered indispensable tools and have been verified by numerous real-world applications and theoretical explorations.
- Research on routine fuzzifications should not be published.
  Nowadays, it is common knowledge validated by a large number of studies that classical concepts, theories, and methods may be fuzzified and that fuzzification results in sound and functional concepts, theories, and methods. There is hence no point in publishing research on fuzzification just for the purpose of demonstrating that a particular fuzzification may be carried out.

Routine fuzzifications are to appear in research works as intermediate steps in solving a given problem or to appear as excercises for students, as is the common practice with other established mathematical methods.

- Assessment of significance should involve the following considerations.
  - Significance from an applicational viewpoint.
    Applicability of fuzzified concepts and methods has been proven by numerous studies which are available in the literature. Hence, to demonstrate that a new fuzzified concept or method may in principle be applied in a situation involving degrees of truth, in particular by using simplistic, toy examples, does not justify significance. To be considered significant from an application viewpoint, the fuzzified method needs to have solid motivations, be tested on non-trivial real-world problems, and if possible compared with established alternative methods.
    Research studies significant from the applicational viewpoint should be submitted to journals with the appropriate scope rather than to fuzzy logic journals. Hence, a study on using a fuzzified method of decision making should be submitted to a journal specializing in decision making, in which the work is more likely to get a proper and adequate review. Such a way of proceeding is obviously overall healthy for fuzzy logic as a research field.
  - Significance from a theoretical viewpoint.
    For the reasons mentioned above, a mere fuzzification of a given classical concept, theory, or method, does not justify its significance. To be considered significant from a theoretical viewpoint, new findings need to be presented, such as novel methods of fuzzification; metaresults on fuzzification, i.e. general results on fuzzification as a method of generalization from the classical setting to the setting of fuzzy logic; and non-trivial properties of the fuzzified concepts and their relationships to established mathematical concepts which are likely to be of a broader interest.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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